Semantically-Aligned Equation Generation for Solving and Reasoning Math Word Problems

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https://github.com/MiuLab/E2EMathSolver
Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?

\[ x = 10 - 1 \times 5 \div 0.5 \]
Prior Work

Non-neural approaches

- Template-based
  (Kushman et al., Upadhyay and Chang)

  \[ x = (\, ? + \, ?) \times \, ? - \, ? \]

  **fill**

  \[ x = (1 + 2) \times 3 - 4 \]

  **Rely on hand-crafted features!**

Deep learning

- Seq2Seq
  (Wang et al., Ling et al.)

  **Problem**

  \[ x = (1+2) \times 3 - 4 \]

  **generate**

  **Does not use the structure of math expression.**

Our model is **end-to-end and structural!**
Overview of the Proposed Model

Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?

\[ x = 10 - 1 \times 5 \div 0.5 \]
Look Again at the Problem

Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?
Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?

\[ x = \left( 10 - 1 \times 5 \right) \div 0.5 \]
Idea: Bridging Symbolic and Semantic Worlds

Symbolic World

\[ \frac{2}{3} + \frac{3}{\infty} \]
\[ 0.999... = 1 \]
\[ \pi \approx 3.14 \]
\[ \sqrt{2} \]
\[ 1 + 2 \cdot 3 \]
\[ (1 - 2) + 3 \]
\[ \frac{5(2 + 2)}{101_2} = 5_{10} \]

Semantic World

MEANING
Each notebook takes $0.5$ and each pen takes $1$. Tom has $10$. How many notebooks can he buy after buying 5 pens?
Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?
Each notebook takes $0.5$ and ...
Semantic Generation for Unknown $x$

* This part is actually done when decoding, but is present at this place for illustration. Check our paper for more information.

Each notebook takes $0.5$ and ...
Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?
Intuition of Using Semantics

Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?

$$x = (10 - \text{Price of a pen} - 5)$$

Number of pens bought.

Price of a pen.
Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?
Equation Generation by Stack Actions

- Stack is used
- The decoder generates stack actions.
- An equation is generated with actions on stack.

\[ x = 10 - 1 \times 5 \div 0.5 \]
Action Selection in Each Step

Encoder

Decoder

stack action
\{+, -, \times, \div, =, Push\}

classifier
Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions:

Action: push
Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions: $x \ 10 \ 1 \ 5$

Action: push
Equation Generation by Stack Actions

Target Equation: \( x = 10 - 1 \times 5 \div 0.5 \)

Generated Actions: \( x \ 10 \ 1 \ 5 \)

Action: \( \times \)
Equation Generation by Stack Actions

Target Equation: \( x = 10 - 1 \times 5 \div 0.5 \)

Generated Actions: \( x\ 10\ 1\ 5\ \times\ 0.5\ \div\ = \)

After many steps...

\[ x = (10 - 1 \times 5) \div 0.5 \]
Each notebook takes $0.5 and each pen takes $1. Tom has $10. How many notebooks can he buy after buying 5 pens?
Experiments

- Dataset: Math23k
- In Chinese
- 23000 math word problems.
- Operators: +, -, ×, ÷
Results

<table>
<thead>
<tr>
<th>Acc.</th>
<th>Retrieval</th>
<th>Template</th>
<th>Generation</th>
<th>Ensemble</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>Retrieval</td>
<td>BLSTM</td>
<td>Seq2Seq w/SNI</td>
<td>Proposed</td>
</tr>
<tr>
<td>60</td>
<td>Self-Attention</td>
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<td></td>
<td>Hybrid</td>
</tr>
<tr>
<td>55</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Ablation Test

Acc.

Char-Based: 66
Word-Based: 66
Word-Based - Semantic: 64
≈ 3%
Word-Based - Gate: 63
Word-Based - Gate - Attention: 62
≈ 2.5%
Word-Based - Gate - Attention - Stack: 60
Self-Attention for Qualitative Analysis

Each notebook takes $0.5 and ...
Self-Attention for Qualitative Analysis

Each notebook takes $0.5$ and ...
The attention focuses on:

- Informative verbs:
  - “gain”, “get”, “fill”, etc.

- Quantifier-related words:
  - “every”, “how many”, etc.
Conclusion

Three main contributions

- **Approach:** equation generation with stack
- **Originality:** automatic extraction of operand semantics
- **Performance:** a SOTA end-to-end neural model on Math23k
Code Available @
https://github.com/MiuLab/E2EMathSolver